International Youth Mathematics Contest 2007

Hongkong Elementary Mathematics International Contest (HEMIC)

Team Competition

Time allowed : 1 hour 30 minutes

Hong Kong : 29 July – 2 August 2007

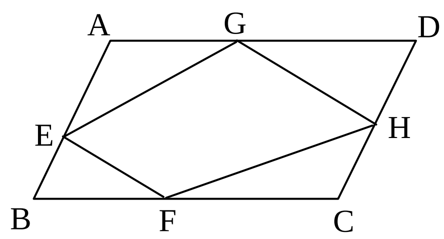
Instructions:

- Write down the team name and the name and candidate number of each team member on the answer sheet.
- Discussion among the team members is allowed.
- Write down all answers on the answer sheet.
- Answer all 10 problems. Problems are in ascending order of level of difficulty. Only NUMERICAL answers are needed.
- Each problem is worth 20 points and the total is 200 points.
- For problems involving more than one answer, points are given only when ALL answers are correct.
- Take π = 3.14 if necessary.
- No calculator or calculating device is allowed.
- Answer the problems with pencil, blue or black ball pen.
- All materials will be collected at the end of the competition.

1. Town A and Town B are connected by a highway which consists of an uphill and a downhill section. A car's speed is 20 km/hr and 35 km/hr for the uphill and downhill sections respectively. It takes 9 hours from A to B but $7\frac{1}{2}$ hours from B to A. What is the downhill distance (in km) from A to B?

2. The houses on one side of a street are numbered using consecutive odd numbers, starting from 1. On the other side, the houses are numbered using consecutive even numbers starting from 2. In total 256 digits are used on the side with even numbers and 404 digits on the side with odd numbers. Find the difference between the largest odd number and the largest even number.

3. As shown in the figure below, *ABCD* is a parallelogram with area of 10. If *AB*=3, *BC*=5, *AE*=*BF*=*AG*=2, *GH* is parallel to *EF*, find the area of *EFHG*.



4. Find the two smallest integers which satisfy the following conditions:
   (1) The difference between the integers is 3.
   (2) In each number, the sum of the digits is a multiple of 11.

5. A four-digit number can be formed by linking two different two-digit prime numbers together. For example, 13 & 17 can be linked together to form a four-digit number 1317 or 1713. Some four-digit numbers formed in this way can be divided by the average of the two prime numbers. Give one possible four-digit number that fulfills the requirement. (Please be reminded that 1317 and 1713 in the example above do not fulfill the requirement, because they are not divisible by 15.)

6. How many prime factors does the number $2 + 2^2 + 2^3 + \ldots + 2^{15} + 2^{16}$ have?

7. A pencil, an easer, and a notebook together cost 100 dollars. A notebook costs more than two pencils, three pencils cost more than four erasers, and three easers cost more than a notebook. How much does each item cost (assuming that the cost of each item is a whole number of dollars)?

8. There are 8 pairs of natural numbers which satisfy the following condition:
The product of the sum of the numbers and the difference of the numbers is 1995.
Which pair of numbers has the greatest difference?

9. A land with a dimension 52 m × 24 m is surrounded by fence. An agricultural scientist wants to divide the land into identical square sections for testing, using fence with total length 1172 m. The sides of the square sections must be parallel to the sides of the land. What is the maximum number of square testing sections that can be formed?

10. Find the total number of ways that 270 can be written as a sum of consecutive positive integers.

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