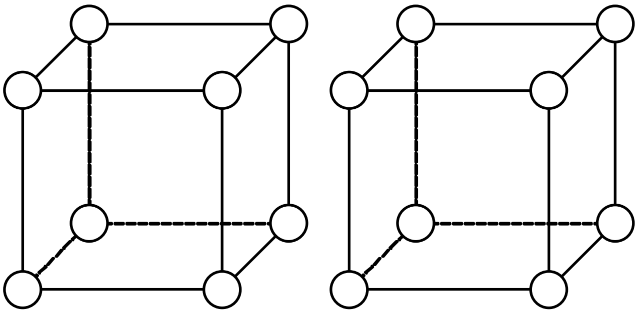
Invitational World Youth Mathematics Intercity Competition 2001

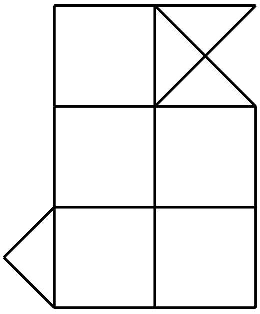
Team Contest

1. Fill in the numbers 1 to 16 on the vertices of two cubes, one number on each vertex with no repetition, such that the sum of the numbers on the four vertices of each face is the same.

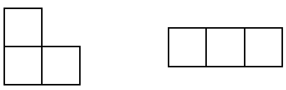


2. Arrange the numbers 1 to 20 in a circle such that the sum of two adjacent numbers is prime.

3. The figure in the diagrm below is a 2×3 rectangle, with one-quarter of the top right square cut off and attached to the bottom left square. Cut the figure along some polygonal line into two identical pieces.

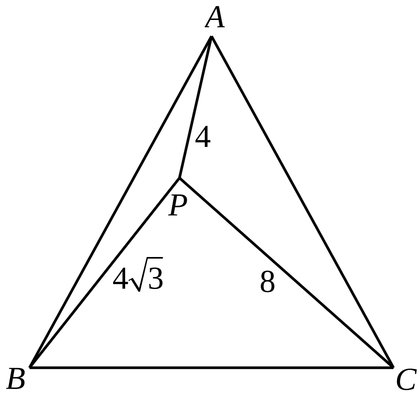


4. A 1×1 cell said to be removable if its removal from an 8×8 square leaves behind a figure which can be tiled by 21 copies of each of the two figures shown in the diagram below. How many removable cells are there in an 8×8 square?



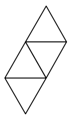
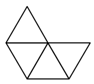
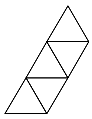
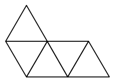
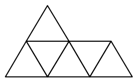
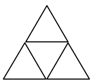
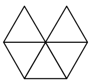
5. The four-digit number 3025 is the square of the sum of the number formed of its first two digits and the number formed of its last two digits, namely, $(30 + 25)^2 = 3025$. Find all other four-digit numbers with this property.

6. P is a point inside an equilateral triangle ABC such that PA=4, PB= $4\sqrt{3}$ and PC=8. Find the area of triangle ABC.



7. The fraction $\frac{1}{4}$ has an interesting property. The numerator is a single-digit number 1 and the denominator is a larger single-digit number 4. If we add the digit 6 after the digit 1 in the numerator n times and add the digit 6 before the digit 4 in the denominator n times also, the fraction $\frac{166\cdots6}{66\cdots64}=\frac{1}{4}$ has the same value. Determine all other fractions with this property, except that the added digit does not have to be 6.

8. There are seven shapes formed of three or four equilateral triangles connected edge-to-edge, as shown in the 2 × 5 chart below.

					
	*1*	*2*	*3*	*4*	*5*
	*6*	*7*	*8*	*9*	*10*

For each of the numbered spaces in the chart, find a figure which can be formed from copies of the shape at the head of the row, as well as from copies of the shape at the head of the column. The pieces may be rotated or reflected. The problem in Space 1 has been solved in the diagram below as an example.