World Youth Mathematics Intercity Competition

Team Contest Time limit: 60 minutes 2008/10/28

Chiang Mai, Thailand

Team: _____ ***Score:*** _____

1. The fraction $\frac{p}{q}$ is in the lowest form. Its decimal expansion has the form 0.*abababab*… The digits *a* and *b* may be equal, except that not both can be 0. Determine the number of different values of *p*.

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2. Cover up as few of the 64 squares in the following 8×8 table as possible so that neither two uncovered numbers in the same row nor in the same column are the same. Two squares sharing a common side cannot both be covered.

6	4	5	7	7	3	3	5
4	8	4	3	6	7	5	1
3	1	5	7	7	7	6	2
7	5	5	8	8	4	2	3
4	5	6	5	8	1	7	3
3	3	3	6	1	8	8	3
1	7	3	2	3	6	4	8
1	6	2	2	4	5	8	7

6	4	5	7	7	3	3	5
4	8	4	3	6	7	5	1
3	1	5	7	7	7	6	2
7	5	5	8	8	4	2	3
4	5	6	5	8	1	7	3
3	3	3	6	1	8	8	3
1	7	3	2	3	6	4	8
1	6	2	2	4	5	8	7

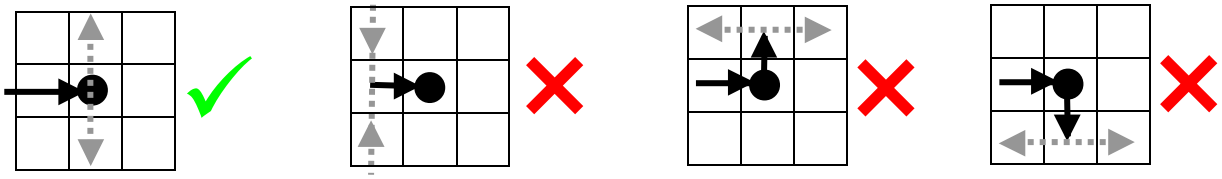
ANSWER :

3. On the following 8×8 board, draw a single path going between squares with common sides so that

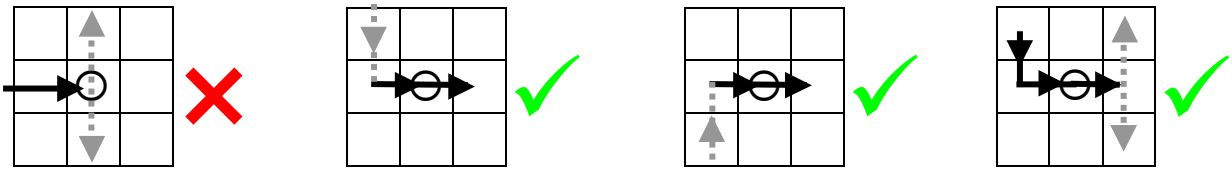
(a) it is closed and not self-intersecting;

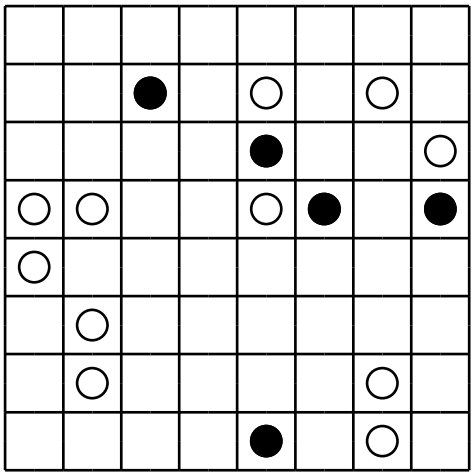
(b) it passes through every square with a circle, though not necessarily every square;

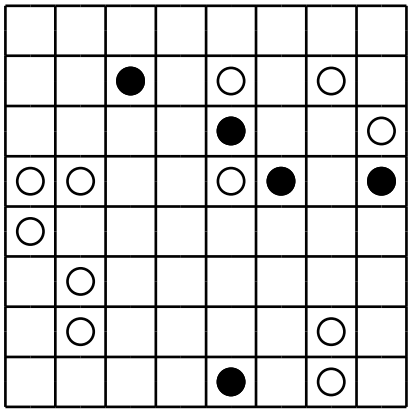
(c) it turns at every square with a black circle, but does not do so on either the square before or the one after;



(d) it does not turn at any square with a white circle, but must do so on either the square before or the one after, or both.







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4. Consider all $a \times b \times c$ boxes where a, b and c are integers such that $1 \le a \le b \le c \le 5$. An $a_1 \times b_1 \times c_1$ box fits inside an $a_2 \times b_2 \times c_2$ box if and only if $a_1 \le a_2$, $b_1 \le b_2$ and $c_1 \le c_2$. Determine the largest number of the boxes under consideration such that none of them fits inside another.

ANSWER : _____

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5. Initially, the numbers 0, 1 and 4 are on the blackboard. Our task is to add more numbers on the blackboard by using the following procedures: In each step, we select two numbers a and b on the blackboard and add the new number $c=ab+a+b$ on the blackboard. What is the smallest number not less than 2008 which can appear on the blackboard after repeating the same procedure for several times?

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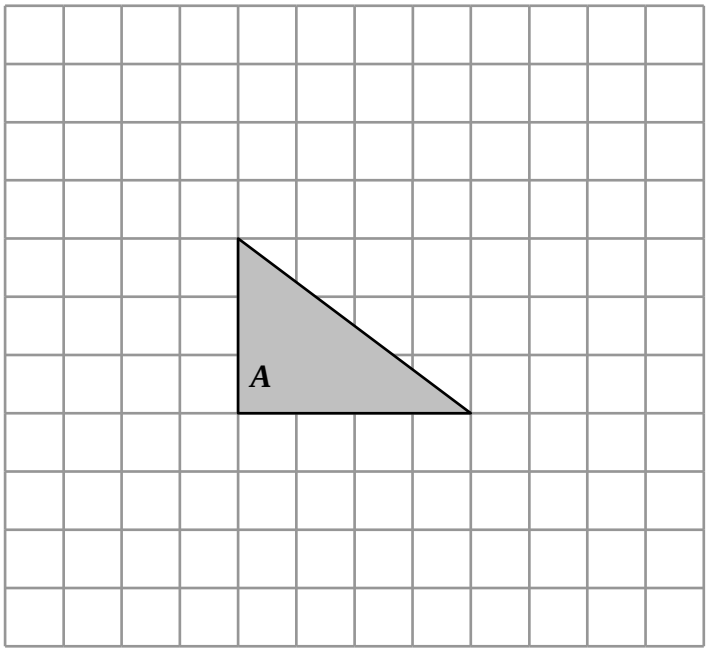
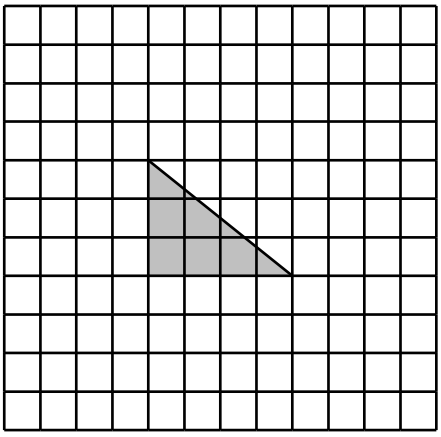
6. Given a shaded triangle as below, find all possible ways of extending one of its sides to a new point so that the resulting triangle has two equal sides. Mark the points of extension on the space given below.





International Mathematics Competition 2008
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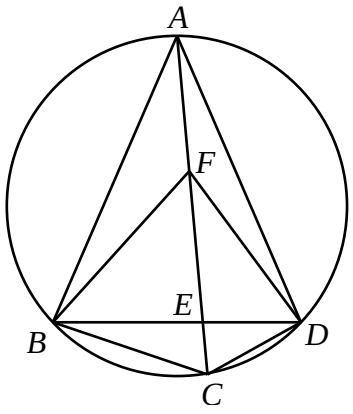
7. *ABCD* is a quadrilateral inscribed in a circle, with *AB*=*AD*. The diagonals intersect at *E*. *F* is a point on *AC* such that $\angle BFC = \angle BAD$. If $\angle BAD = 2\angle DFC$, determine $\frac{BE}{DE}$.



Team: _____ ***Score:*** _____

8. How many five-digit numbers are there that contain the digit 3 at least once?

ANSWER : _____

Team: _____ ***Score:*** _____

9. Among nine identically looking coins, one of them weighs a grams, seven of them b grams each and the last one c grams, where $a<b<c$. We wish to determine whether $a+c<2b$, $a+c=2b$ or $a+c>2b$ using only an unmarked beam balance four times.

ANSWER : _____

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10. Determine the sum of all positive integers n such that

$$1+n+\frac{n(n-1)}{2}+\frac{n(n-1)(n-2)}{6}=2^k \quad \text{for some positive integer } k.$$

ANSWER : _____

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