Individual Contest

Time limit: 120 minutes

Instructions:

- Do not turn to the first page until you are told to do so.
- Remember to write down your team name, your name and contestant number in the spaces indicated on the first page.
- The Individual Contest is composed of two sections with a total of 120 points.
- Section A consists of 12 questions in which blanks are to be filled in and only **ARABIC NUMERAL** answers are required. For problems involving more than one answer, points are given only when **ALL** answers are correct. Each question is worth 5 points. There is no penalty for a wrong answer.
- Section B consists of 3 problems of a computational nature, and the solutions should include detailed explanations. Each problem is worth 20 points, and partial credit may be awarded.
- You have a total of 120 minutes to complete the competition.
- No calculator, calculating device, watches or electronic devices are allowed.
- Answers must be in pencil or in blue or black ball point pen.
- All papers shall be collected at the end of this test.

English Version

Team:______________ Name:__________________ No.: _____________ Score:__________

For Juries Use Only

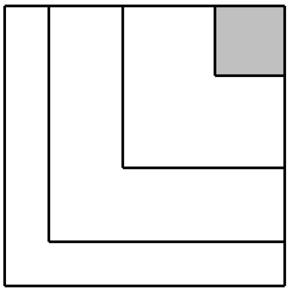
No.	Section A												Section B			Total	Sign by Jury
	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3		
Score																	
Score																	

Section A.
In this section, there are 12 questions. Fill in the correct answer in the space provided at the end of each question. Each correct answer is worth 5 points.

1. In this problem, different letters stand for different digits and identical letters stand for the same digit. The three-digit number *ABB* is 25 less than the three-digit number *CDC*. The number *ABBCDC* is the square of a positive integer. What is that positive integer?

Answer : ________

2. A house 30 m by 30 m is at the north-east corner of a farm 120 m by 120 m. The house owner wanted to divide the remaining part with two V-shaped fences into three V-shaped plots of equal area, as shown in the diagram below. Each segment of the fence is perpendicular to a side of the farm, and the two segments of the same fence have equal length. What is the length, in m, of the shorter fence?



Answer : ________ m

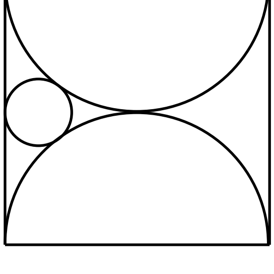
3. In how many ways can the six letters in the word MOUSEY be arranged in a row without containing either the word YOU or the word ME? For example, the word MOUSEY itself is such an arrangement.

Answer : ______ ways

4. How many pairs (a, b) of positive integers are there such that $a \le b$ and $2\left(\sqrt{\frac{15}{a}}+\sqrt{\frac{15}{b}}\right)$ is an integer?

Answer : ______ pairs

5. The diagram below shows a square of side length 80 cm. It contains two semicircles which touch each other at the centre of the square, and a small circle which is tangent to the square and both semicircles. What is the radius, in cm, of the small circle?

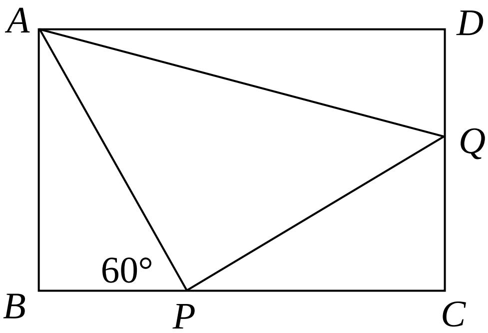


Answer : ______ cm

6. What is the greatest length of a block of consecutive positive integers each of which can be expressed as the sum of the squares of two positive integers?

Answer : __________

7. *APQ* is a right isosceles triangle inscribed in a rectangle *ABCD*, with the vertex *P* of the right angle on *BC* and *Q* on *CD*. If *BP* = 1 cm and $\angle APB = 60°$, what is the area, in cm^2, of triangle *ADQ*?



Answer : ______ cm^2

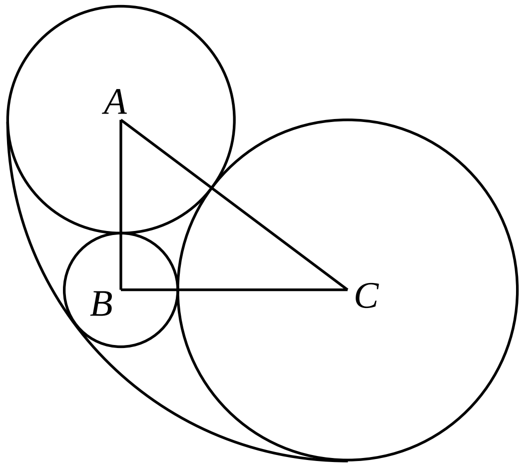
8. Lea has a diamond ring, a gold ring and an ivory ring. She put them on her right hand. Each ring can be on any of the five fingers. When there are two or three rings on the same finger, if the order in which they are put is different, that counts as a different way. What is the number of different ways for Lea to put on these three rings?

Answer : ______ ways

9. Let *a*, *b* and *c* be positive integers. If the greatest common divisor of $b + c$, $c + a$ and $a + b$ is *k* times the greatest common divisor of *a*, *b* and *c*, what is the maximum value of *k*?

Answer : __________

10. In triangle *ABC*, *BC* = 4 cm, *CA* = 5 cm and *AB* = 3 cm. Three circles with respective centres *A*, *B* and *C* are pairwise tangent. *A* fourth circle is tangent to those three circles and contains all of them, as shown in the diagram below. What is the radius, in cm, of the fourth circle?

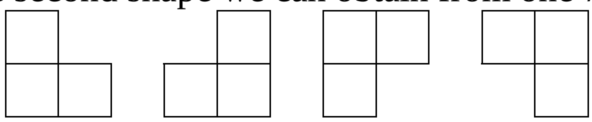


Answer : ______ cm

11. The positive integers $a < b$ are such that $\dfrac{a+b}{2}$ and $\sqrt{ab}$ are positive integers consisting of the same two digits in reverse order. What is the minimum value of a?

Answer : _________

12. A factory produces metal plates in two shapes. The first shape is a 2×2 square. The second shape, as shown in the diagram below, is a 2×2 square with one of the four cells missing. These two shapes metal plates are cut from 7×7 metal sheets, and none of the 49 cells may be wasted. What is the minimum number of metal plates of the second shape we can obtain from one 7×7 metal sheet?



Answer : _________

Section B.

Answer the following 3 questions, and show your detailed solution in the space provided after each question. Each question is worth 20 points.

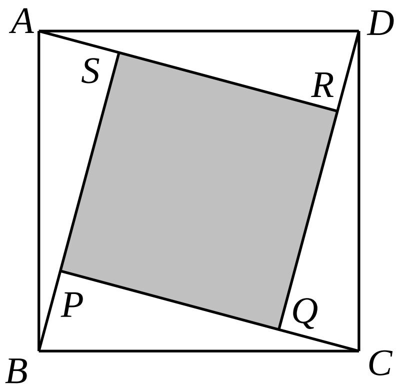
1. Consider the expression

$$1+\frac{1}{2}+\cdots+\frac{1}{n}+\left(1+\frac{1}{2}+\cdots+\frac{1}{n}\right)^2+\left(\frac{1}{2}+\cdots+\frac{1}{n}\right)^2+\cdots+\left(\frac{1}{n}\right)^2.$$

Starting from the second bracket, the sum inside is obtained by removing the first term from the sum in the preceding bracket. What is the value of this expression when $n = 2013$?

Answer : _________

2. In the diagram below, *ABCD* is a square, and $\angle PCB = \angle QDC = \angle RAD = \angle SBA$. If the area of *ABCD* is twice the area of *PQRS*, what is the measure, in degrees, of $\angle PCB$?



$^\circ$

Answer : ________

3. There are eight coins in a row, all showing heads. In each move, we can flip over two adjacent coins provided that they are both showing heads or both showing tails. How many different patterns of heads and tails can we obtain after a number of moves?

Answer : ___________