2014 Korea International Mathematics Competition

21~26 July, 2014, Daejeon City, Korea

Invitational World Youth Mathematics Intercity Competition

Individual Contest

Time limit: 120 minutes

Instructions:

- Do not turn to the first page until you are told to do so.
- Remember to write down your team name, your name and contestant number in the spaces indicated on the first page.
- The Individual Contest is composed of two sections with a total of 120 points.
- Section A consists of 12 questions in which blanks are to be filled in and only **ARABIC NUMERAL** answers are required. For problems involving more than one answer, points are given only when **ALL** answers are correct. Each question is worth 5 points. There is no penalty for a wrong answer.
- Section B consists of 3 problems of a computational nature, and the solutions should include detailed explanations. Each problem is worth 20 points, and partial credit may be awarded.
- You have a total of 120 minutes to complete the competition.
- No calculator, calculating device, electronic devices or protractor are allowed.
- Answers must be in pencil or in blue or black ball point pen.
- All papers shall be collected at the end of this test.

English Version

Team:_____________ **Name:**_________________ **No.:** _____________ **Score:**_________

For Juries Use Only

No.	Section A												Section B			Total	Sign by Jury
	1	2	3	4	5	6	7	8	9	10	11	12	1	2	3		
Score																	
Score																	

Section A.

In this section, there are 12 questions. Fill in the correct answer in the space provided at the end of each question. Each correct answer is worth 5 points.

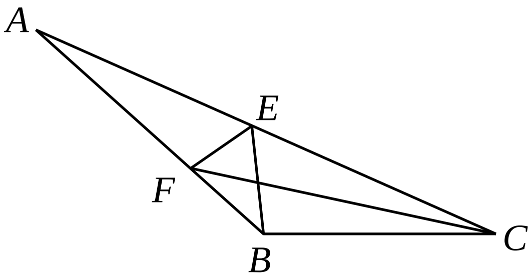
1. A two-digit number x is twice another two-digit number y. The sum of the digits of x is equal to one of the digits of y and the difference between the digits of x is equal to the other digit of y. What is the maximum value of y?

Answer : 17

2. What is the difference when 1234567891 × 1234567896 × 1234567898 is subtracted from 1234567899 × 1234567894 × 1234567892?

Answer : 24

3. In triangle ABC, $\angle CAB = 18°$ and $\angle BCA = 24°$. E is a point on CA such that $\angle CEB = 60°$ and F is a point on AB such that $\angle AEF = 60°$. What is the measure, in degrees, of $\angle BFC$?



Answer : 30 °

4. The real numbers x, y and z are such that $x - 7y + 8z = 4$ and $8x + 4y - z = 7$. What is the maximum value of $x^2 - y^2 + z^2$?

Answer : 1

5. In triangle ABC, $\angle CAB = \angle BCA = 45°$. L is the midpoint of BC and P is a point on CA such that BP is perpendicular to AL. If $CP = \sqrt{2}$ cm, what is the length, in cm, of AB?

Answer : 3 cm

6. If x, y and z are three consecutive positive integers such that $\frac{x}{y} + \frac{y}{z} + \frac{z}{x} + \frac{y}{x} + \frac{x}{z} + \frac{z}{y}$ is an integer, what is the value of $x + y + z$?

Answer : 6

7. The real numbers x and y are such that $x^3 + y^3 = 1957$ and $(x + y)(x + 1)(y + 1) = 2014$. What is the value of $x + y$?

Answer : 19

8. In the parallelogram *ABCD*, $\angle BAD = 60°$. *E* is the midpoint of *BC* and *F* is the midpoint of *CD*. *BD* intersects *AE* at *M* and *AF* at *N*. If *AB* = 15 cm and *AD* = 8 cm, what is the length, in cm, of *MN*?

Answer : $\frac{13}{3}$ cm

9. In how many different ways can eight children (including Anna, Bert and Cody) be seated at a round table, if Anna must be next to Bert and Anna must not be sitting next to Cody? Seating arrangements in the same clockwise order are not considered to be different.

Answer : 1200 ways

10. What is the largest integer *n* such that $(\frac{21}{n}-2)^2 - 2(\frac{21}{n}-2) = n + 42$?

Answer : −7

11. Consider all the positive integers up to 10000000000 in which every digit is 0 or 2. What is the total number of 0s among their digits?

Answer : 4097

12. *A*, *B*, *C* and *D* are four points on a circle in that cyclic order. If $AD = BD = 50\sqrt{3}$ cm, *AC* = 106.8 cm and $\angle CAD = 30°$, what is the length, in cm, of *BC*?

Answer : 43.2 cm

Section B.

Answer the following 3 questions, and show your detailed solution in the space provided after each question. Each question is worth 20 points.

1. On the blackboard are written the reciprocals of the first 2014 positive integers. In

a move, we may erase two numbers and replace them by the sum of their sum and their product. Eventually, only one number remains. What is the maximum value of this number?

【Solution】

Note that we are replacing a and b by $a+b+ab=(a+1)(b+1)-1$. On a second blackboard, copy all the numbers from the first blackboard but increasing each number by 1. Then we are replacing $a+1$ and $b+1$ by their product. Since the numbers on the second blackboard are 2, $\frac{3}{2}$, $\frac{4}{3}$, …, $\frac{2015}{2014}$ initially, the final number must be the product of them all, which is 2015. It follows that there is only one possible value of the final number on the first blackboard, namely, 2015 −1 = 2014.

Answer ： 2014

【Marking Scheme】

A student may deduce the answer from observing the pattern: 1 = 1, $1+\frac{1}{2}+1\times\frac{1}{2}=2$, $2+\frac{1}{3}+2\times\frac{1}{3}=3$, and so on. This is worth **10 marks**.

2. In triangle ABC, $\angle ABC=\angle ACB=30°$. O is a point such that OA = OC = 1 cm and OB = 2 cm. Moreover, $\angle CAB=\angle CAO+\angle OAB$. What is the length, in cm, of BC?

Answer **:** ____________ cm

3. The Tower of Daejeon consists of 7 disks of different sizes, arranged in increasing order of size from top to bottom on the first of three pegs. The task is to transfer the tower from the first peg to the third peg. In each move, the top disk from a peg may be transferred to the top of an adjacent peg. Thus transfers directly from the first peg to the third peg or vice versa are forbidden. A larger disk may never be on

top of a smaller disk. What is the minimum number of moves required?

Answer : ___________